

Diferencijalne j-ne I-og reda

Def Jna oblika  $F(x, y, y', \dots, y^{(n)}) = 0, y = y(x)$  naziva se difer. jednačinom.

Dif- jne koje se razdvajaju promenjivima

$y' = f(x) \cdot g(y), f, g \in C, y = y(x)$

$\frac{dy}{dx} = f(x) \cdot g(y), g(y) \neq 0$

$\frac{dy}{g(y)} = f(x) dx \Rightarrow \int \frac{dy}{g(y)} = \int f(x) dx$

$G(y) = F(x) + C$

1)  $y' = 2xy$

$\frac{dy}{dx} = 2xy$

$\frac{dy}{y} = 2x dx$

$\int \frac{dy}{y} = \int 2x dx$

$\ln|y| = 2 \frac{x^2}{2} + C$

$\ln|y| = x^2 + C$

$|y| = e^{x^2 + C}$

$y = \pm e^{x^2 + C}, \pm e^C = A$

$y = Ae^{x^2}$

2)  $y' = 1 + x + y + xy$

$\frac{dy}{dx} = 1 + x + y(1+x)$

$\frac{dy}{dx} = (1+x)(1+y)$

$\frac{dy}{1+y} = (1+x) dx$

$\int \frac{dy}{1+y} = \int (1+x) dx$

$\ln|1+y| = x + \frac{x^2}{2} + C$

$|1+y| = e^{x + \frac{x^2}{2} + C}$

$y = \pm e^{x + \frac{x^2}{2} + C} - 1$

3)  $x^3 y^2 y' = 1 - x^2$

$xy \cdot \frac{dy}{dx} = 1 - x^2$

$y dy = \frac{1 - x^2}{x} \cdot dx$

$\int y dy = \int \frac{1 - x^2}{x} dx$

$\int y dy = \int \frac{dx}{x} - \int x dx$

$\frac{y^2}{2} = \ln|x| - \frac{x^2}{2} + C$

$y^2 = 2(\ln|x| - \frac{x^2}{2} + C)$

$y = \pm \sqrt{2(\ln|x| - \frac{x^2}{2} + C)}$



4)  $y' \cdot \operatorname{tg} x = y$

$$\frac{dy}{dx} = \frac{y}{\operatorname{tg} x}$$

$$\frac{dy}{y} = \frac{dx}{\operatorname{tg} x}$$

$$\int \frac{dy}{y} = \int \frac{dx}{\operatorname{tg} x}$$

$$\ln|y| = \int \frac{dx \cdot \cos x}{\sin x}$$

$\int \sin x = t$   
 $\cos x dx = dt \rightarrow$

$$\ln|y| = \int \frac{dt}{t}$$

$$\ln|y| = \ln|t| + c$$

$$\ln|y| = \ln|\sin x| + c$$

$$|y| = e^{\ln|\sin x| + c} = A \cdot |\sin x|$$

$$y = |\sin x| \cdot A$$

5)  $(1+x)y' + 1+y = 0$

$$(1+x) \frac{dy}{dx} = -1-y$$

$$\frac{dy}{-1-y} = \frac{dx}{1+x}$$

$$-\int \frac{dy}{1+y} = \int \frac{dx}{1+x}$$

$$\ln|1+y| = -\ln|1+x| + c$$

$$|1+y| = e^{-\ln|1+x| + c}$$

$$y = \pm e^{-\ln|1+x| + c} = \frac{A}{1+x}$$

6)  $xy' - y = y^3$

$$x \cdot \frac{dy}{dx} = y^3 + y$$

$$\frac{x}{dx} = \frac{y^3 + y}{dy}$$

$$\frac{dx}{x} = \frac{dy}{y^3 + y}$$

$$\int \frac{dx}{x} = \int \frac{dy}{y(y^2+1)} \quad (1)$$

$$I_1 = \int \frac{dy}{y(y^2+1)}$$

$$\frac{1}{y(y^2+1)} = \frac{A}{y} + \frac{By+C}{y^2+1}$$

$$1 = Ay^2 + A + By^2 + Cy$$

$$A+B=0 \rightarrow B=-1$$

$$A=1$$

$$C=0$$

$$I_1 = \int \frac{dy}{y} + \int \frac{-y}{y^2+1} dy = \int \frac{dy}{y} - \frac{1}{2} \int \frac{2y}{y^2+1} dy$$

$$= \ln|y| - \frac{1}{2} \int \frac{dt}{t} = \ln|y| - \frac{1}{2} \ln|y^2+1|$$

$$\ln|x| = \ln|y| - \frac{1}{2} \ln|y^2+1| + c$$

$$\ln|x| = \ln \frac{y}{\sqrt{y^2+1}} + c$$

$$\frac{y}{\sqrt{y^2+1}} = |x| \cdot e^c$$



$$y' = 10^{x+y}$$

$$\frac{dy}{dx} = 10^x \cdot 10^y$$

$$\frac{dy}{10^y} = 10^x dx$$

$$\int \frac{dy}{10^y} = \int 10^x dx$$

$$\int 10^{-y} dy = \int 10^x dx$$

$$-\frac{10^{-y}}{\ln 10} = \frac{10^x}{\ln 10} + C$$

$$8) y' = \frac{1+y^2}{(1+x^2)xy}$$

$$\frac{dy}{dx} = \frac{1+y^2}{y} \cdot \frac{1}{(1+x^2)x}$$

$$y \frac{dy}{1+y^2} = \frac{1}{(1+x^2)x} dx$$

$$\int \frac{y dy}{1+y^2} = \int \frac{dx}{x(1+x^2)}$$

$$\frac{1}{2} \ln|1+y^2| = \ln|x| - \frac{1}{2} \ln|x^2+1| + C$$

$$\sqrt{1+y^2} = \frac{x}{\sqrt{x^2+1}} e^C$$

9a)  $\sqrt{1-x^2} dy - x\sqrt{1-y^2} dx = 0$  (uogd n.)

d)  $(y^2 + xy^3)y' + x^2 - yx^2 = 0$  (uogd n.)

b)  $\frac{dy}{\sqrt{x}} = \frac{3dx}{\sqrt{y}}$  (uogd n.)

$$I_1 = -ye^{-y} + \int e^{-y} dy =$$

c)  $yy' = xe^{x^2+y^2}$  (uogd n.)

$$= -ye^{-y} - e^{-y} = \underline{e^{-y}(y-1)}$$

10)  $(1+e^x)yy' = e^y, y(0)=0$

$$I_2 = \int \frac{dx}{1+e^x}, \quad e^x = t, \quad x = \ln t, \quad dx = \frac{dt}{t}$$

~~By:~~  $(1+e^x)y \cdot \frac{dy}{dx} = e^y$

$$\int \frac{y}{e^y} dy = \int \frac{dx}{1+e^x}$$

$$I_2 = \int \frac{dt}{t(1+t)}$$

$$I_1 = \int \frac{y dy}{e^y}$$

$$\frac{1}{t(1+t)} = \frac{A}{t} + \frac{B}{1+t} \dots$$

$y = u, \quad \frac{dy}{e^y} = du$

$$I_2 = \ln|t| - \ln|1+t| + C =$$

$y = au, \quad v = \int \frac{dy}{e^y} = \underline{\underline{-e^{-y}}}$

$$= \underline{\underline{\ln e^x - \ln(1+e^x) + C}}$$



$$I_1 = I_2 \Rightarrow -e^{-f(y+1)} = \ln e^x - \ln(1+e^x) + c$$

$$g(0) = 0 \quad -e^0(0+1) = \ln e^0 - \ln(1+e^0) + c$$

$$-1 = \ln 1 - \ln 2 + c$$

$$-1 = -\ln 2 + c$$

$$\boxed{c = \ln 2 - 1}$$

$$-e^{-f(y+1)} = \ln e^x + \ln(1+e^x) + \ln 2 - 1$$

91) a)  $y' + y^2 = 1, y(0) = 0$ , b)  $e^x y' = xy^2, y(1) = 1$

Homogene dif. j-ne

(1)  $y' = g\left(\frac{y}{x}\right)$  sugere  $\frac{y}{x} = u, u = u(x)$

$$y = u \cdot x \Rightarrow y' = u' \cdot x + u$$

(1)  $\Rightarrow u' \cdot x + u = g(u)$

$$\frac{du}{dx} \cdot x = g(u) - u$$

$$\int \frac{du}{g(u) - u} = \int \frac{dx}{x}$$

1)  $y' = \frac{xy + y}{x}$

$$y' = 1 + \frac{y}{x}, \frac{y}{x} = u$$

$$u' \cdot x + u = 1 + u$$

$$u' \cdot x = 1$$

$$x \frac{du}{dx} = 1$$

$$\int du = \int \frac{dx}{x}$$

$$u = \ln|x| + c$$

$$\frac{y}{x} = \ln|x| + c$$

$$y = x(\ln|x| + c)$$

2)  $x^2 y' = xy + x^2 y^2 \quad | : x^2$

$$y' = \frac{y}{x} + 1 + \left(\frac{y}{x}\right)^2, \quad u = \frac{y}{x}, y = u \cdot x$$

$$u' \cdot x + u = u + 1 + u^2$$

$$u' \cdot x = 1 + u^2$$

$$\frac{du}{dx} \cdot x = 1 + u^2$$

$$\int \frac{dx}{x} = \int \frac{du}{1+u^2}$$

$$\ln|x| = \arctg u$$

$$\arctg \frac{y}{x} = \ln|x| + c$$



3)  $(x^2 + y^2) dx - 2xy dy = 0 \quad /: x^2$   
 $(1 + (\frac{y}{x})^2) dx - 2 \frac{y}{x} dy = 0$   
 $(\frac{y}{x}) = u \Rightarrow y = xu$

$(1 + u^2) dx - 2u dy = 0$   
 $1 + u^2 = 2u \left(\frac{dy}{dx}\right) = y'$   
 $1 + u^2 = 2u \cdot (u'x + u)$   
 $1 + u^2 = 2uu'x + 2u^2$   
 $1 - u^2 = 2u \frac{du}{dx} \cdot x$

$\int \frac{dx}{x} = \int \frac{2u du}{1 - u^2} \quad | \quad 1 - u^2 = t$   
 $\ln|x| + C = -\ln|1 - u^2| + C$   
 $\ln|x| = -\ln|1 - (\frac{y}{x})^2| + C$

$\ln|\ln(u)| = \ln|x| + C$

$\ln|\ln \frac{y}{x}| = \ln|x| \cdot e^C \Rightarrow \ln \frac{y}{x} = |x| \cdot e^C$

4)  $xy' = y(1 + \ln \frac{y}{x})$   
 $y' = \frac{y}{x} (1 + \ln \frac{y}{x})$   
 $\frac{y}{x} = u$

$u'x + u = u(1 + \ln u)$   
 $x \frac{du}{dx} + u = u + u \ln u$   
 $\frac{x du}{dx} = u \ln u$   
 $x du = u \ln u dx$   
 $\left( \frac{du}{u \ln u} = \int \frac{dx}{x} \right)$

$\ln u = u \rightarrow I_1 = \int \frac{du}{u} = \ln|u|$   
 $\frac{du}{u} = du$   
 $= \ln|\ln|u||$

5) 2a y:

a)  $3y^2 - 2xyy' + x^2 = 0$  (Jelimo)  $/: x^2$

$3 \frac{y^2}{x^2} - 2y' \frac{y}{x} + 1 = 0$   
 $\frac{y}{x} = u$

b)  $(y + \sqrt{x^2 + y^2}) dx - x dy = 0$  (moja sv.)  $/: x$

$y = ux$

c)  $xy^2 dy = (x^3 + y^3) dx$  (moja sv.)

$3u^2 - 2(u'x + u)u + 1 = 0$

$(\frac{y}{x} + \sqrt{1 + (\frac{y}{x})^2}) dx - y dy = 0 \quad /: dx$

$u^2 - 2 \frac{du}{dx} xu + 1 = 0$

$\int \frac{2u du}{u^2 + 1} = \int \frac{dx}{x}$

$u^2 + 1 = 2 \left( \frac{xu du}{dx} \right) \quad /: u du$



## Lineare dif.-gln

Opšti oblik <sup>lin</sup> dif.-gln prvog reda:  $y' + yP(x) = Q(x)$

$$\frac{y'}{y} + P(x) - \frac{Q(x)}{y} = 0$$

$$\frac{y'}{y} = P(x) = \frac{z'}{z}, \quad z = z(x), \quad \frac{z'}{z} - \frac{Q(x)}{y} = 0 \quad (*)$$

Integralujemo dobijamo:

$$\ln y - \ln z = - \int P(x) dx \Rightarrow \ln \frac{y}{z} = - \int P(x) dx \Rightarrow$$

$$y = z e^{-\int P(x) dx}$$

$$u(x) \Rightarrow z = \int Q(x) e^{\int P(x) dx} dx + C$$

$$\text{Konačno } y = e^{-\int P(x) dx} \left( \int Q(x) e^{\int P(x) dx} dx + C \right).$$

$$1) (1-x^2)y' + 2xy - 4x = 0 \quad | : (1-x^2)$$

$$y' + \frac{2xy}{1-x^2} - \frac{4x}{1-x^2} = 0$$

$$y' + \frac{2x}{1-x^2} y = \frac{4x}{1-x^2}$$

$$P(x) = \frac{2x}{1-x^2}, \quad Q(x) = \frac{4x}{1-x^2}$$

$$y = e^{-\int P(x) dx} \left( \int Q(x) e^{\int P(x) dx} dx + C \right)$$

$$\int \frac{2x}{1-x^2} dx = \int \frac{-dt}{t} = -\ln |1-x^2|$$

$1-x^2 = t$   
 $-2x dx = dt$



$$\int \frac{4x}{1-x^2} e^{-\ln|1-x^2|} dx = \int \frac{4x}{1-x^2} \cdot \frac{1}{1-x^2} dx$$

$$\begin{aligned} (1-x^2) &= t \\ -2x dx &= dt \\ x dx &= -\frac{dt}{2} \end{aligned} \Rightarrow \int \frac{4 \cdot \frac{-dt}{2}}{t^2} = -2 \int \frac{dt}{t^2} =$$

$$= -2 \cdot \frac{t^{-1}}{-1} + c = \underline{\underline{\frac{2}{1-x^2} + c}}$$

$$y = (1-x^2) \left( \frac{2}{1-x^2} + c \right) = \underline{\underline{2 + c(1-x^2)}}$$

2)  $xy' - y + \ln x = 0$  i. x  $\int y^{(1)} = ?$   $\int -\frac{1}{x} dx = -\ln|x| \checkmark$

$$y' - \frac{y}{x} + \frac{\ln x}{x} = 0$$

$$y' - \frac{1}{x} y = -\frac{\ln x}{x}$$

$$P(x) = -\frac{1}{x}, \quad Q(x) = -\frac{\ln x}{x}$$

$$\int -\frac{\ln x}{x} \cdot e^{-\ln|x|} =$$

$$= \int -\frac{\ln x}{x} \cdot \frac{1}{x} dx =$$

$$= -\int \frac{\ln x}{x^2} dx$$

$$y = e^{-\int P(x) dx} \left( \int Q(x) e^{\int P(x) dx} + c \right)$$

$$I = \int \frac{\ln x}{x^2} dx = \begin{cases} \ln x = u & \frac{dx}{x^2} = dv \\ \frac{dx}{x} = du & \int \frac{dx}{x^2} = -\frac{1}{x} \end{cases}$$

$$\int \frac{dx}{x^2} = -\frac{1}{x}$$

$$\Rightarrow I = -\frac{1}{x} \ln x + \int \frac{1}{x} \frac{dx}{x} = \underline{\underline{-\frac{1}{x} \ln x + \frac{1}{x} + c}}$$

$$y = e^{\ln|x|} \left( +\frac{1}{x} \ln x + \frac{1}{x} + c \right) = \underline{\underline{\ln x + 1 + cx}}$$

$$2 = 0 + 1 \cdot c \cdot 1 \rightarrow c + 1 = 2$$

$$\boxed{c=1}$$



$$3) y' + y \operatorname{tg} x = \frac{1}{\cos x}$$

$$P(x) = \operatorname{tg} x, \quad Q(x) = \frac{1}{\cos x}$$

$$y = e^{-\int P(x) dx} \left( \int Q(x) e^{\int P(x) dx} dx + C \right)$$

$$\int \operatorname{tg} x dx = \int \frac{\sin x}{\cos x} dx, \quad \begin{cases} \cos x = t \\ -\sin x dx = dt \end{cases} = \int -\frac{dt}{t} = -\frac{\ln |t|}{1} = -\ln |\cos x|$$

$$\int \frac{1}{\cos x} e^{-\ln |\cos x|} dx = \int \frac{1}{\cos^2 x} dx = \operatorname{tg} x + C$$

$$y = \cos x (\operatorname{tg} x + C) = \underline{\sin x + C \cos x}$$

$$4) y' + y \operatorname{tg} x - \sin 2x = 0 \quad P(x) = \operatorname{tg} x, \quad Q(x) = \sin 2x \quad (\text{Jelmo})$$

$$y = e^{-\int P(x) dx} \left( \int Q(x) e^{\int P(x) dx} dx + C \right)$$

$$5) (\sin^2 y + x \operatorname{ctg} y) y' = 1$$

Postupakajme  $x = x(y), \quad y' = \frac{1}{x'}$

$$1 = x' \cdot y' \Rightarrow y' = \frac{1}{x'}$$

$$\Rightarrow (\sin^2 y + x \operatorname{ctg} y) \cdot \frac{1}{x'} = 1 \rightarrow$$

$$x' - x \operatorname{ctg} y = \sin^2 y, \quad \begin{cases} P(y) = -\operatorname{ctg} y \\ Q(y) = \sin^2 y \end{cases}$$

$$\int \operatorname{tg} x dx = -\ln |\cos x|$$

$$\int \sin 2x e^{\ln \frac{1}{\cos x}} dx =$$

$$= \int 2 \sin x \cos x \frac{1}{\cos x} dx =$$

$$= \underline{2(-\cos x)}$$

$$\underline{x = \sin y (-\cos y + C)}$$

$$y = \cos x (2 - \cos x) + C$$

$$y' - y \operatorname{ctg} x = \sin^2 x$$

$$P(x) = -\operatorname{ctg} x$$

$$Q(x) = \sin^2 x$$

$$y' - y \operatorname{ctg} x = \sin^2 x$$



# Beemulyeva j-na

$$\text{Opšti oblik } y' + P(x)y = Q(x)y^d$$

$d=0$  i  $d=1 \rightarrow$  lin. nehom. dif. j-na i lin. homog. dif. j-na.

$$d \neq 0 \wedge d \neq 1 \rightarrow z = y^{1-d} \Rightarrow$$

$$\frac{1}{1-d} \frac{dz}{dx} + P(x)z = W(x)$$

$y = uv$  ili varij. konstanti.

$$1) \quad xy' + y - y^2 \ln x = 0$$

$$xy' + y = y^2 \ln x \quad /: x \quad P(x) =$$

$$y' + \frac{1}{x}y = y^2 \cdot \frac{\ln x}{x}, \quad P(x) = \frac{1}{x}, \quad Q(x) = \frac{\ln x}{x}, \quad d=2$$

$$\text{Smjena: } z = y^{1-d} \Rightarrow z = y^{1-2} = \frac{1}{y} \Rightarrow y = \frac{1}{z}$$

$$y' = -\frac{1}{z^2} \cdot z'$$

$$-\frac{z'}{z^2} + \frac{1}{x} \cdot \frac{1}{z} = \frac{1}{z^2} \frac{\ln x}{x} \quad / \cdot (-z^2)$$

$$z' - z \cdot \frac{1}{x} = -\frac{\ln x}{x}$$

$$z = e^{-\int -\frac{dx}{x}} \left( \int -\frac{\ln x}{x} \cdot e^{\int -\frac{dx}{x}} dx + C \right) \dots$$

$$z = Cx + 1 + \ln|x|$$

$$; \int \frac{dx}{x} = \ln|x|$$

$$z = y^{-1} \Rightarrow y = \frac{1}{Cx + 1 + \ln|x|} \quad \left| \int \frac{\ln x}{x} \cdot e^{\ln x^{-1}} dx =$$

$$= -\int \frac{d \ln x}{x^2} dx = \left[ \ln x = u \right. \\ \left. \frac{dx}{x^2} = du \right]$$



$$2) \quad xy' - 4y = x\sqrt{y} \quad /: x\sqrt{y}$$

$$\frac{y'}{\sqrt{y}} - \frac{4}{x}\sqrt{y} = x$$

$$\sqrt{y} = z \Rightarrow z' = \frac{1}{2\sqrt{y}} y'$$

$$\frac{2\sqrt{y} \cdot z'}{\sqrt{y}} - \frac{4}{x}z = x$$

$$2z' - \frac{4}{x}z = x$$

$$z' - \frac{2}{x}z = \frac{x}{2} \quad , P(x) = -\frac{2}{x} \quad Q(x) = \frac{x}{2}$$

$$\int -\frac{2}{x} dx = -2 \ln|x|$$

$$\int Q(x) e^{\int P(x) dx} dx = -\int \frac{x}{2} \frac{1}{x^2} dx = -\frac{1}{2} \ln|x|$$

$$z = e^{-\int P(x) dx} \left( \int Q(x) e^{\int P(x) dx} dx + C \right) =$$

$$= e^{\ln x^2} \left( -\frac{1}{2} \ln|x| + C \right) = \frac{1}{2} x^2 \ln|x| + x^2 C$$

$$y = z^2 = \frac{1}{4} x^4 \ln^2|x| + x^4 C^2 + x^4 \ln|x| C$$

$$3) \quad y' + y = xy^3 \quad /: y^3 \quad P(x) = 1, \quad Q(x) = x, \quad \alpha = 3$$

$$z = y^{1-3} = y^{-2} \rightarrow z = \frac{1}{y^2}, \quad z' = -\frac{2}{y^3} y'$$

~~$$\frac{y'}{y^3} + \frac{1}{y^2} = x$$~~

$$\frac{y'}{y^3} + \frac{1}{y^2} = x$$

$$-\frac{z'}{2} + z = x$$

$$z' - 2z = -2x \quad , P(x) = -2, \quad Q(x) = -2x$$

$$/: x \quad y' - \frac{4}{x}y = x y^{1/2}$$

$$z = y^{1/2} \Rightarrow$$

$$y = z^2 \Rightarrow y' = 2z \cdot z'$$

$$2z z' - \frac{4}{x} \cdot z^2 = x \cdot z \quad /: z$$

$$z' - \frac{2}{x}z = \frac{x}{2}$$

$$P(x) = -\frac{2}{x}, \quad Q(x) = \frac{x}{2}$$

$$\int -\frac{2}{x} dx = -2 \ln|x|$$

$$\int \frac{x}{2} e^{\ln x^{-2}} = \int \frac{1}{2x} dx = \frac{1}{2} \ln|x| + C$$



$$1) y' + y \operatorname{tg} x = \sin 2x$$

$$P(x) = \operatorname{tg} x, \quad Q(x) = \sin 2x$$

$$y = e^{-\int P(x) dx} \left( \int Q(x) e^{\int P(x) dx} dx + C \right)$$

$$\int \operatorname{tg} x dx = \int \frac{\sin x}{\cos x} dx = \int \cos x = t \quad \left. \begin{array}{l} \cos x = t \\ -\sin x dx = dt \end{array} \right\} = \int \frac{-dt}{t} = -\ln|t| = -\ln|\cos x|$$

$$\int \sin 2x e^{-\ln|\cos x|} dx = \int \sin 2x \frac{1}{\cos x} dx =$$

$$= \int 2 \sin x \cos x \frac{1}{\cos x} dx = 2 \int \sin x dx = \underline{\underline{-2 \cos x}}$$

$$y = e^{\ln|\cos x|} (-2 \cos x + C) = \underline{\underline{\cos x (-2 \cos x + C)}}$$



$$(y^2 + xy^2) y' + x^2 - yx^2 = 0$$

$$(y^2 + xy^2) \frac{dy}{dx} = yx^2 - x^2$$

$$y^2(1+x) \frac{dy}{dx} = x^2(y-1)$$

$$\int \frac{y^2}{y-1} dy = \int \frac{x^2}{1+x} dx$$

$$\frac{y^2}{y-1} = y+1 + \frac{1}{y-1}$$

$$\frac{y^2}{y-1}$$

$$\int y dy + \int dy + \int \frac{dy}{y-1} =$$

$$= \frac{y^2}{2} + y + \ln|y-1|$$

$$\frac{x^2}{1+x} = (x+1) = x-1 + \frac{1}{x+1}$$

$$\frac{-x}{-x-1}$$

$$\int \frac{x^2+1-1}{1+x} dx =$$

$$\int x dx - \int dx + \int \frac{dx}{x+1} = \int \frac{(x-1)(x+1)}{x+1} dx +$$

$$+ \int \frac{dx}{1+x}$$

$$= \frac{x^2}{2} - x + \ln|x+1|$$

Homogene

$$3y^2 - 2xyy' + x^2 = 0 \quad | : x^2$$

$$\frac{3y^2}{x^2} - 2 \frac{y}{x} y' + 1 = 0$$

$$\frac{y}{x} = u \Rightarrow y = ux$$

$$y' = u'x + u$$

$$3u^2 - 2u(u'x + u) + 1 = 0$$

$$3u^2 - 2uu'x - 2u^2 + 1 = 0$$

$$u^2 = 2uu'x - 1 \quad | : 2u$$

$$u = 2u'x$$

$$u^2 + 1 = 2uu'x \quad | : u$$

$$\frac{u^2+1}{u} = 2u'x$$

$$u' = \frac{du}{dx}$$

$$\frac{u^2+1}{u} = 2 \frac{du}{dx} x$$

$$\int \frac{2u du}{u^2+1} = \int \frac{dx}{x}$$

$$u^2+1 = t$$

$$2u du = dt$$

$$\int \frac{dt}{t} = \ln|t| + c \Rightarrow \ln|u^2+1| = \ln|x| + c$$



$$1) \sqrt{1-x^2} dy - x\sqrt{1-y^2} dx = 0$$

$$\sqrt{1-x^2} dy = x\sqrt{1-y^2} dx$$

$$\frac{\sqrt{1-x^2}}{x} dy = \sqrt{1-y^2} dx$$

$$\frac{dy}{\sqrt{1-y^2}} = \frac{x dx}{\sqrt{1-x^2}}$$

$$\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{x dx}{\sqrt{1-x^2}}$$

$$\arcsin y = \int \frac{x dx}{\sqrt{1-x^2}}$$

$$1-x^2 = t$$

$$-2x dx = dt \Rightarrow$$

$$x dx = -\frac{dt}{2}$$

$$I = -\int \frac{dt}{2\sqrt{t}} = -\frac{1}{2} \int \frac{dt}{\sqrt{t}} =$$

$$= -\frac{1}{2} \int dt t^{-1/2} = -\frac{1}{2} \frac{t^{-1/2+1}}{-1/2+1} =$$

$$= -\frac{1}{2} \cdot \frac{\sqrt{t}}{\frac{1}{2}} = -\sqrt{t}$$

$$\arcsin y = -\sqrt{1-x^2} + c$$

$$2) yy' = xe^{x^2+y^2}$$

$$y \frac{dy}{dx} = xe^{x^2} \cdot e^{y^2}$$

$$\int \frac{y}{e^{y^2}} dy = \int xe^{x^2} dx$$

$$y^2 = t \Rightarrow 2y dy = dt$$

$$\frac{1}{2} \int \frac{dt}{e^t} = \frac{1}{2} \int e^{-t} dt = -\frac{1}{2} e^{-y^2}$$

$$x^2 = m \Rightarrow 2x dx = dm$$

$$x dx = \frac{dm}{2}$$

$$\frac{1}{2} \int e^m dm = \frac{1}{2} e^{x^2}$$

$$-\frac{1}{2} e^{-y^2} = \frac{1}{2} e^{x^2} + c \quad | \cdot 2$$

$$e^{-y^2} = -e^{x^2} + c_1 =$$

$$-y^2 = \ln(e^{x^2} + c_1)$$

$$y^2 = -\ln(-e^{x^2} + c_1)$$



## Tablica integrala

$$\int dx = x + C$$

$$\int x^a dx = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C \quad (a > 0, a \neq 1)$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{dx}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{dx}{\sqrt{1+x^2}} = \operatorname{Arsh} x + C = \ln(x + \sqrt{1+x^2}) + C$$

$$\int \frac{dx}{\sqrt{x^2-1}} = \operatorname{Arch} x + C = \ln|x + \sqrt{x^2-1}| + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C \quad (a > 0)$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

$$\int \frac{dx}{\sqrt{a^2+x^2}} = \ln(x + \sqrt{a^2+x^2}) + C \quad (a > 0)$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln|x + \sqrt{x^2-a^2}| + C \quad (a > 0)$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left| \frac{x+a}{x-a} \right| + C \quad (a > 0)$$

$$\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \operatorname{ch} x dx = \operatorname{sh} x + C$$

$$\int \operatorname{sh} x dx = \operatorname{ch} x + C$$

$$\int \frac{dx}{\operatorname{ch}^2 x} = \operatorname{th} x + C$$

$$\int \frac{dx}{\operatorname{sh}^2 x} = -\operatorname{cth} x + C$$